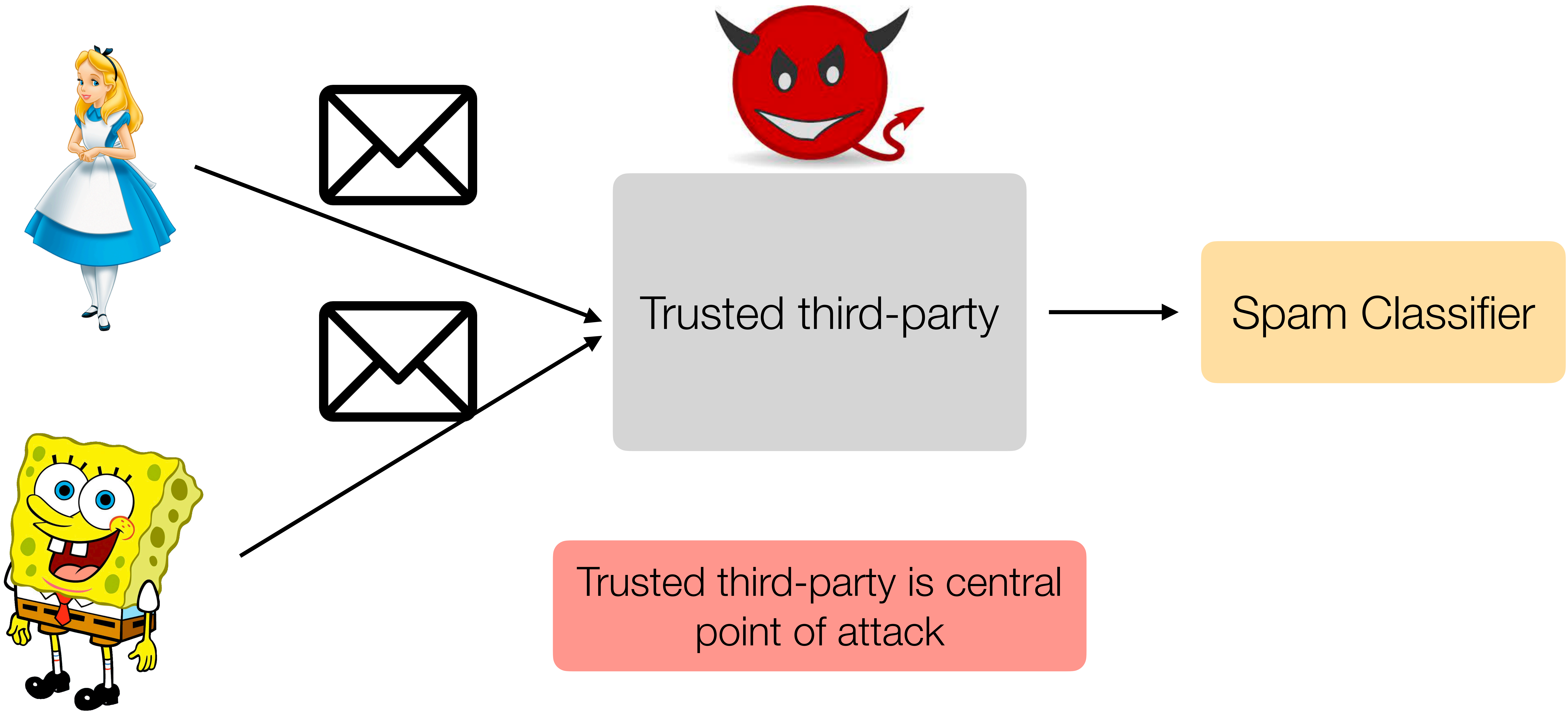
CS 350S: Privacy-Preserving Systems

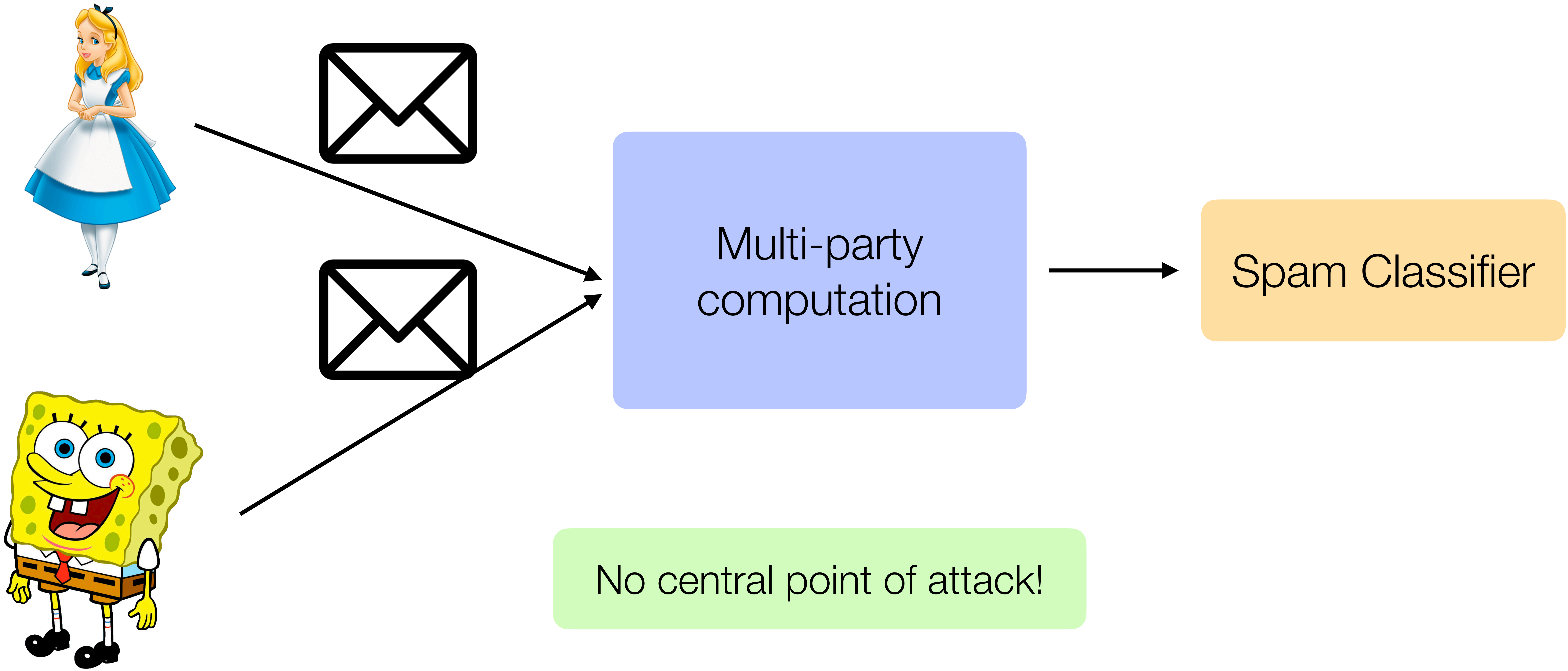
Multi-party computation II

Defining MPC



Defining MPC

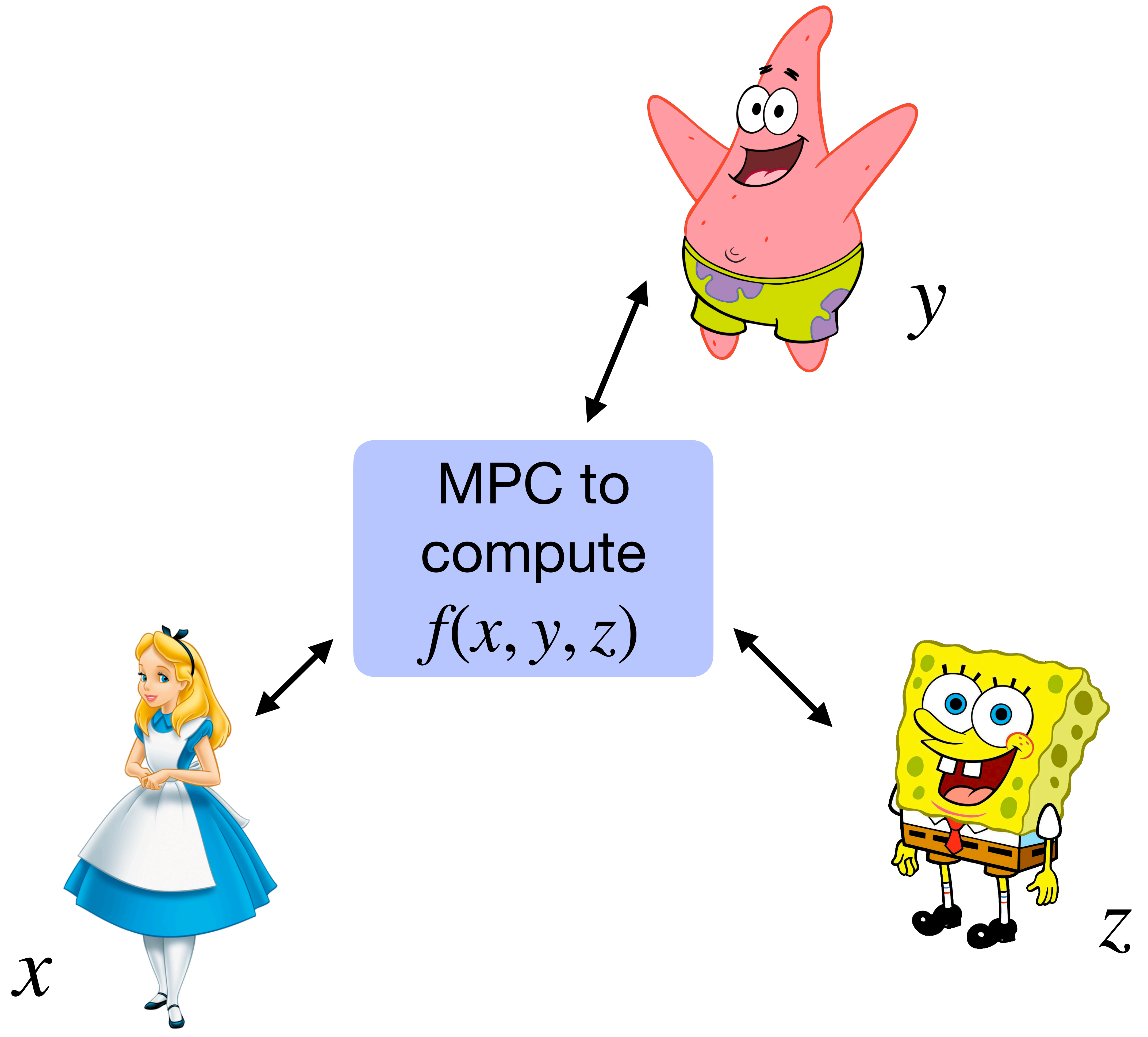
(Informal) Any computation that can be performed with a trusted third party can be securely computed *without* one!



Defining MPC

Parties with inputs x, y, z that want to jointly compute the function $f(x, y, z)$

- Assume encrypted, authenticated channels between parties
- Generalize to any number of parties



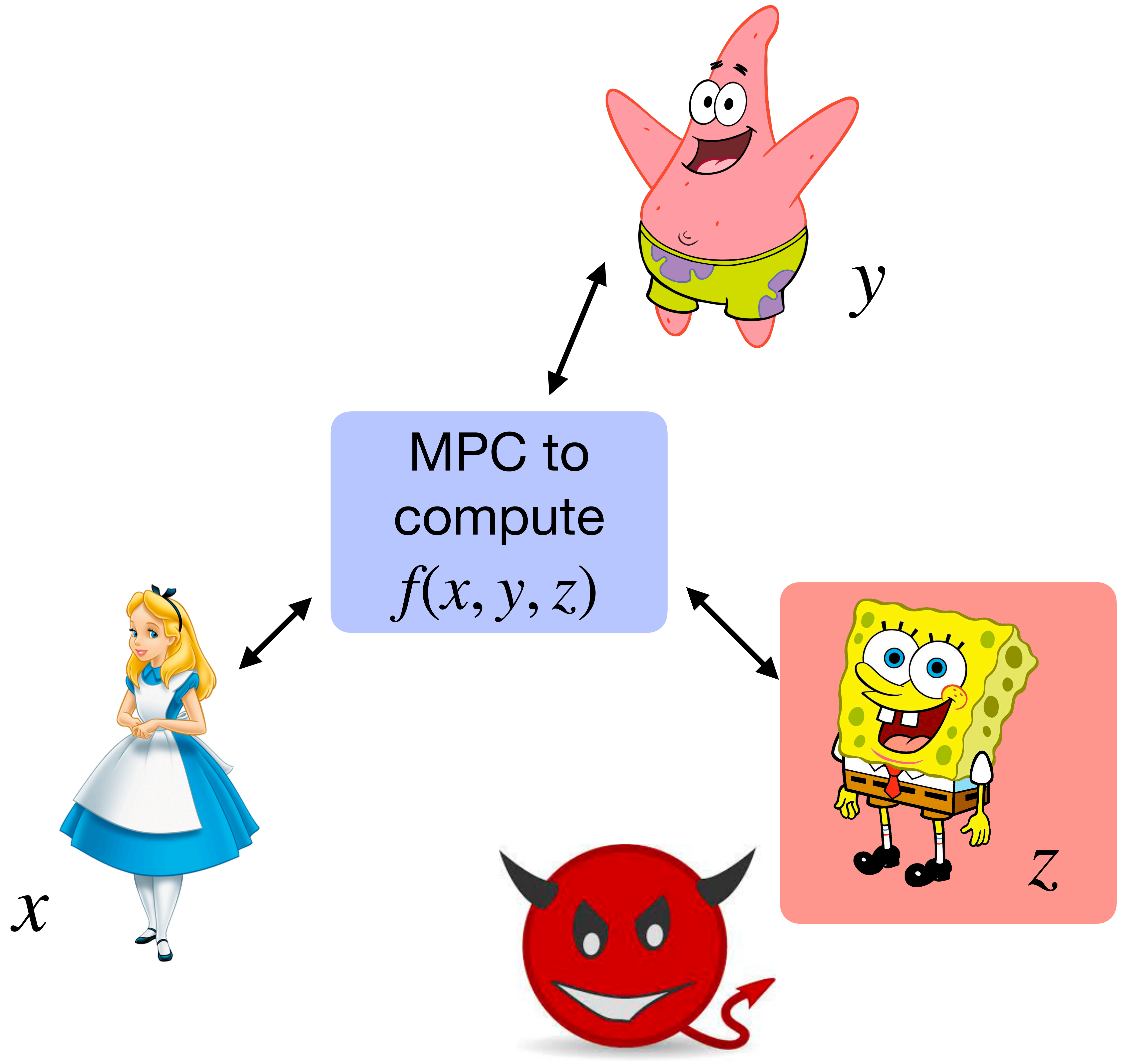
Defining MPC

Parties with inputs x, y, z that want to jointly compute the function $f(x, y, z)$

- Assume encrypted, authenticated channels between parties
- Generalize to any number of parties

Defends against attacker that compromises a subset of the parties

- Exact security properties depends on MPC protocol



Two adversary models in MPC

Semihonest: The corrupted parties *follow the protocol specification*. After the protocol completes, they look at the transcript and try to extract info about the honest parties' input.

Malicious: The corrupted parties may *arbitrarily deviate from the protocol specification* to learn extra info about the honest parties' inputs or trick them into producing the wrong output.

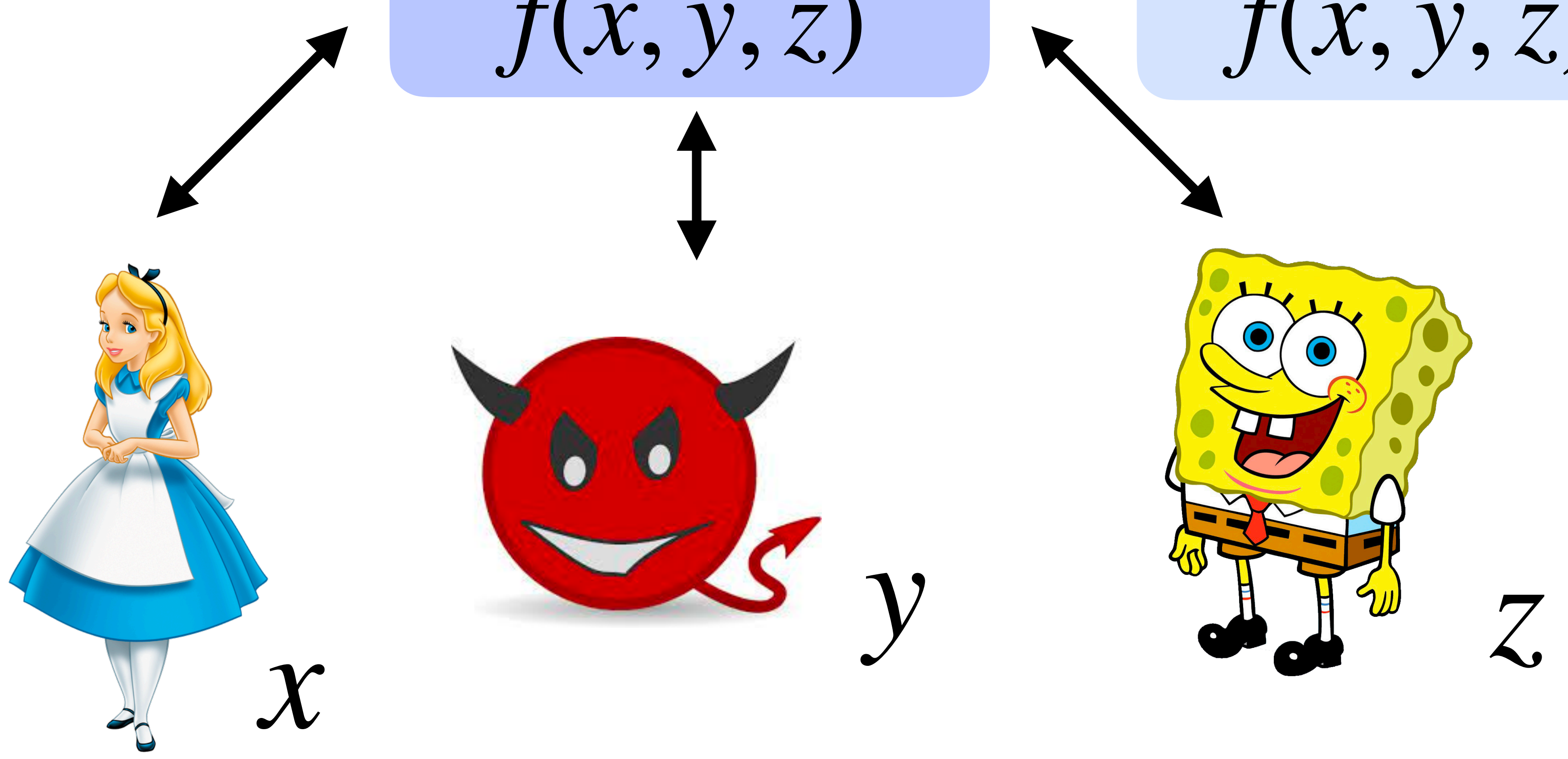
Defining semihonest security

Informally: Anything the adversary learns in an execution of the MPC, it could also have learned if all the parties were interacting with a trusted third party

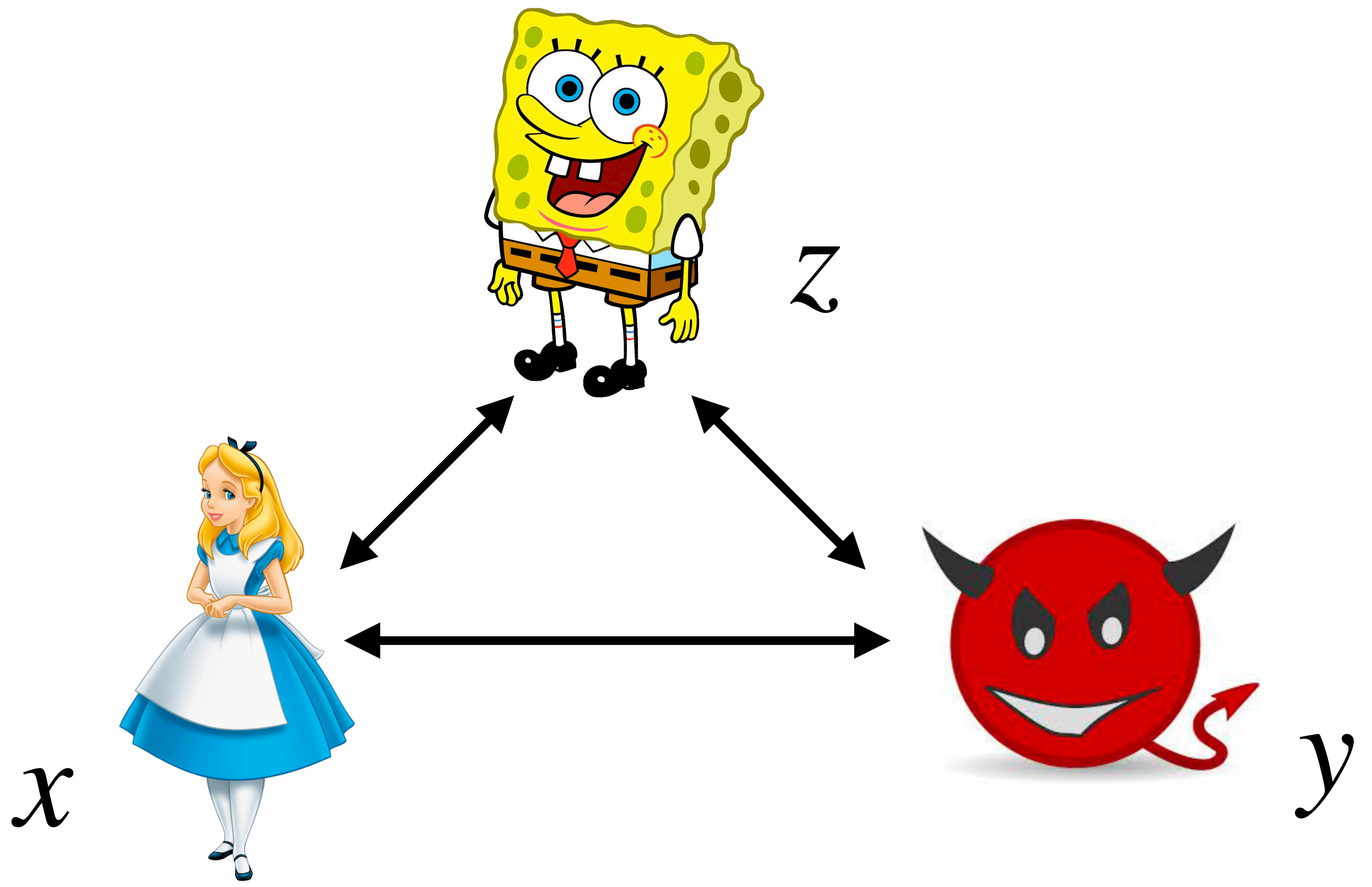
Ideal world

Trusted third-party for $f(x, y, z)$

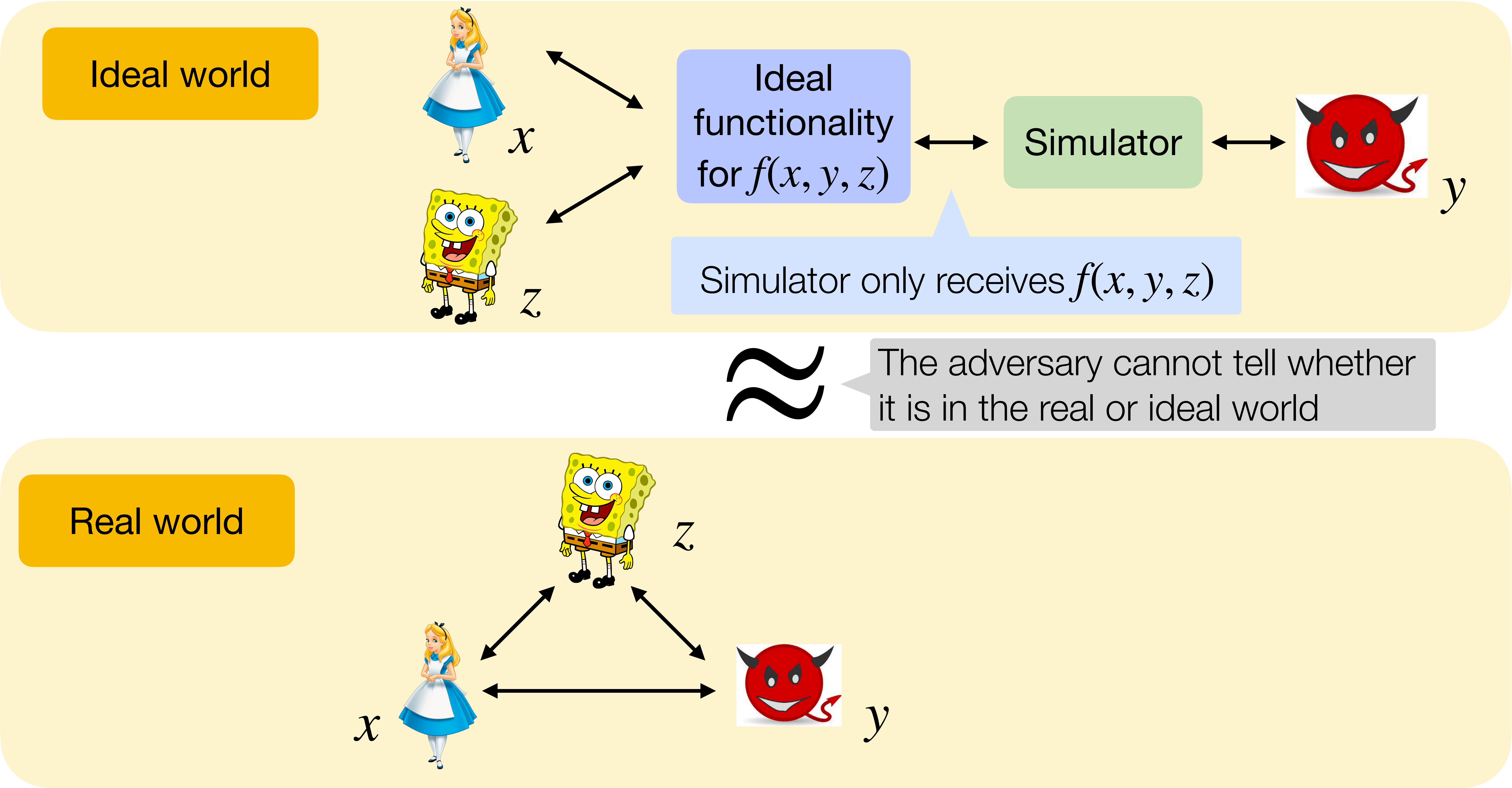
In ideal world, attacker learns
- Input of corrupted party y
- Output of computation $f(x, y, z)$



Real world



Defining semihonest security



Outline

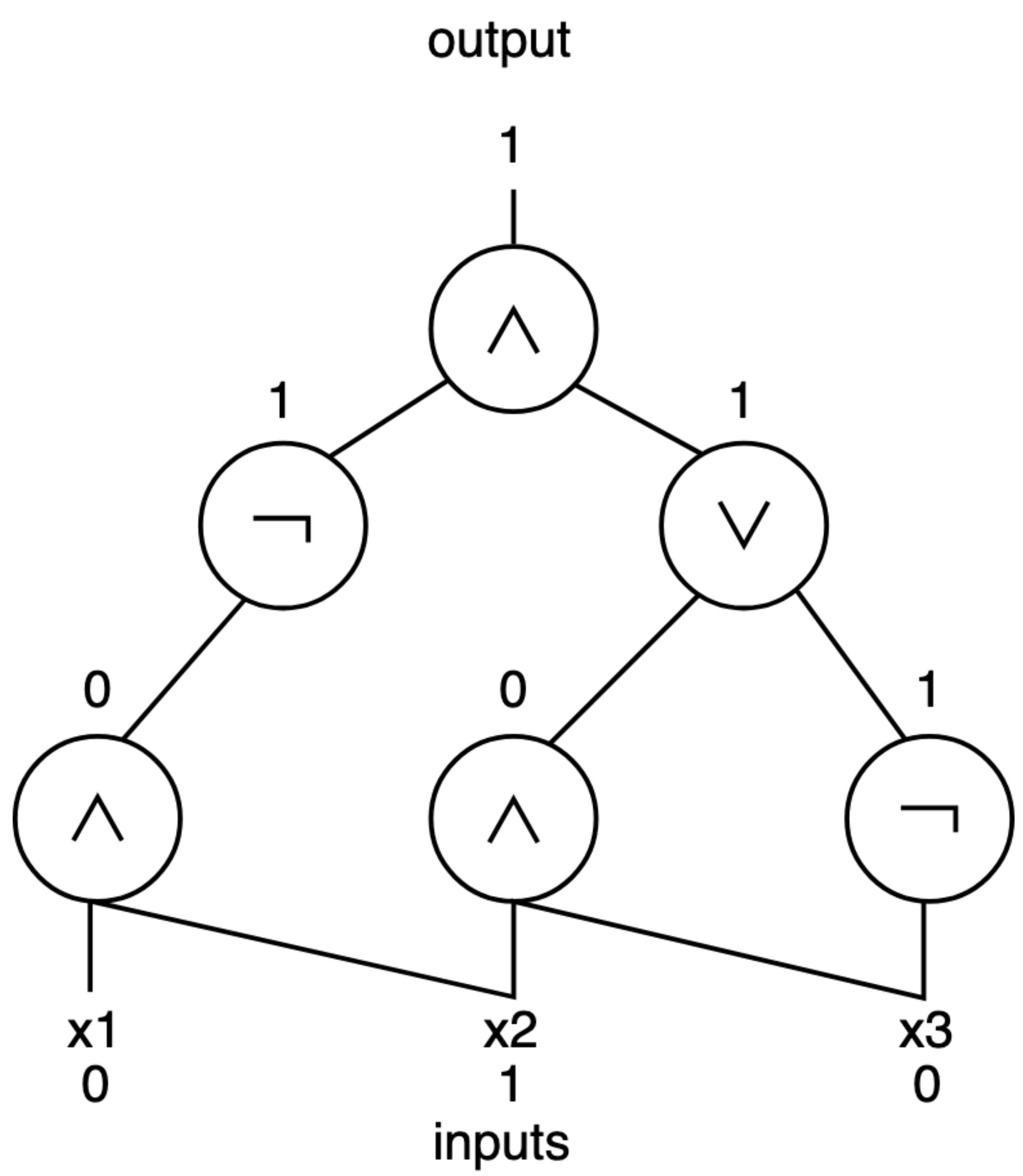
1. **Garbled circuits**
2. MAGE
3. Applications of MPC
4. Student presentation

Yao's garbled circuits [Yao82]

Multi-party computation for boolean circuits

Starting point: evaluating 1 AND gate

Next: generalize to a function f



Starting point: Garbled AND gate [Yao82]

Want: privately compute an AND gate for both parties' inputs (α, β)

2 parties: Garbler and Evaluator with inputs (b_0, b_1)

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0	0	0
0	1	0
1	0	0
1	1	1

Starting point: Garbled AND gate

1. For each of (α, β), the garbler samples a pair of keys $(k_L^0, k_L^1), (k_R^0, k_R^1)$

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Why does the *garbler* not learn more than the output?

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Why does the *garbler* not learn more than the output?

Garbler only gets the oblivious transfer messages and the output from the evaluator

k_L^1	k_R^1	$c_{10} = E(k_L^1, E(k_R^1, 1))$

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Why does the *evaluator* not learn more than the output?

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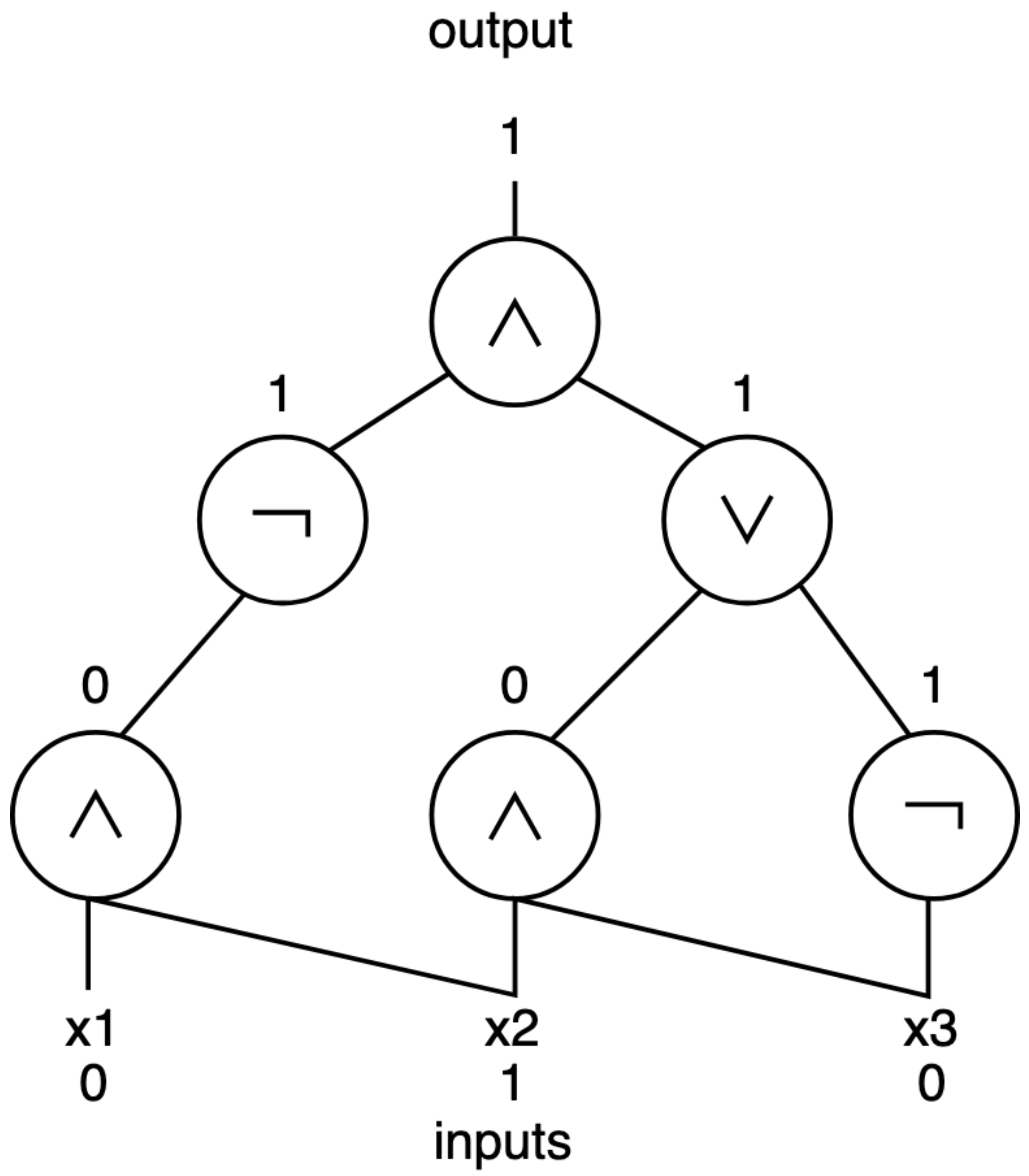
Why does the *evaluator* not learn more than the output?

- Ciphertexts: private via semantic security
- $k_L^{b_0}$: randomly generated key, evaluator does not learn value of b_0
- Oblivious transfer messages hide retrieved value

Yao's garbled circuits

Now: extend the warm-up to a general 2PC protocol for arbitrary boolean circuits

Say that evaluator has $x \in \{0,1\}^n$, garbler has $y \in \{0,1\}^n$, want to compute $f(x, y)$



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Why is the protocol correct, i.e., output $f(x, y)$?

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7. The evaluator has the key for each of the $2n$ input wires and keys to get $f(x, y)$
8. The evaluator sends the output $f(x, y)$ to the garbler

Why is the protocol correct, i.e., output $f(x, y)$?

- At each gate, evaluator gets the key for the correct output of the gate.
- Given the correct input keys, the evaluate gets correct shares of 0/1 at the output gates
- Oblivious transfer ensures that evaluator gets the right keys

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Why is the protocol private?

- The garbler only sees oblivious transfer messages and the output
- The evaluator only sees the permuted ciphertexts, oblivious transfer messages, and the output

Outline

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2. **MAGE**
3. Applications of MPC
4. Student presentation

MAGE motivation

Garbled circuit: Each bit corresponds to 4 ciphertexts

- Blowup in space

Homomorphic encryption: Ciphertexts require more storage than plaintext values

Size of memory limits the types of computations that are practical to run

MAGE key idea

[Kumar, Culler, Popa]

Observation: memory access patterns of MPC programs are deterministic

- Data values are not visible, and so memory accesses are data-independent (obliviousness)

Key idea: Obliviousness makes it possible to compute the set of memory accesses in advance

- Can prefetch data ahead of time

Traditional memory management vs. MAGE

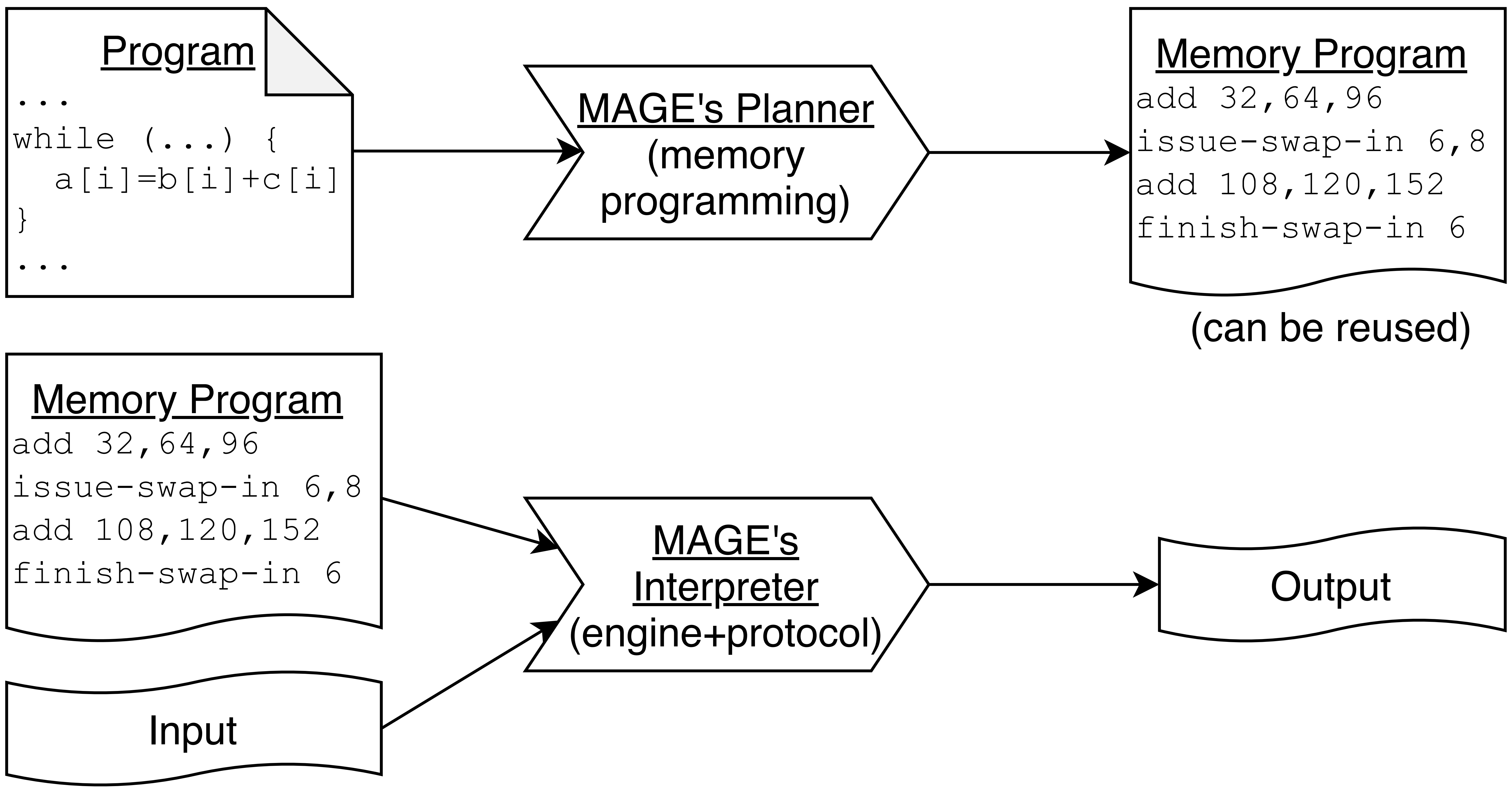
Virtual memory

- Don’t know which memory address will be accessed next
- Page in memory from disk as needed

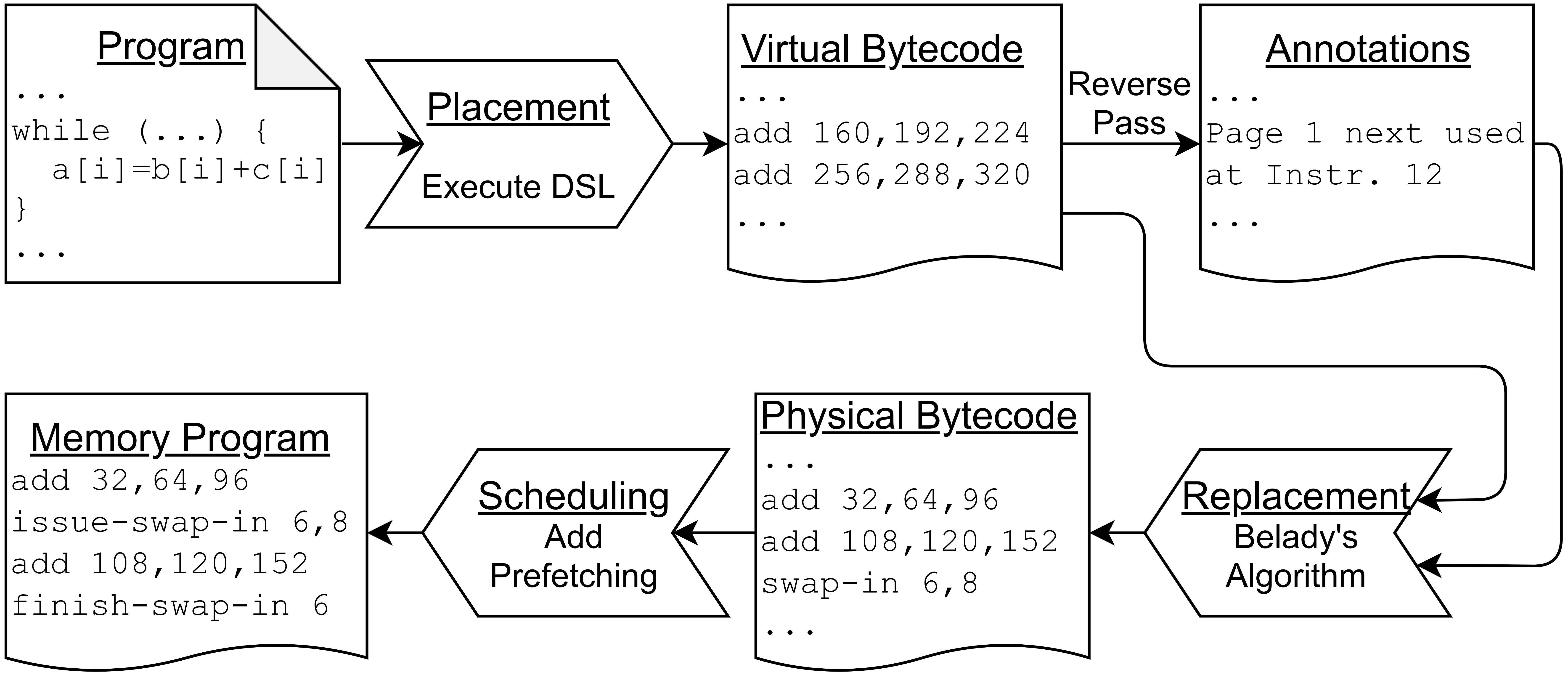
MAGE

- Know exactly which memory address will be accessed next
- Prefetch memory before it is needed
- Dramatically reduces overheads of virtual memory

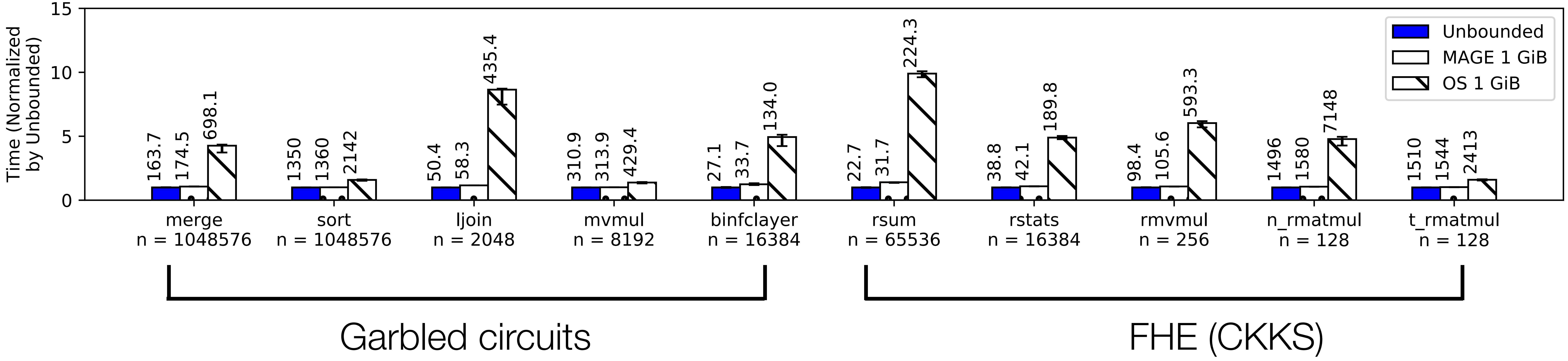
MAGE design steps



MAGE planner



MAGE evaluation



Outline

1. Garbled circuits
2. MAGE
3. **Applications of MPC**
4. Student presentation

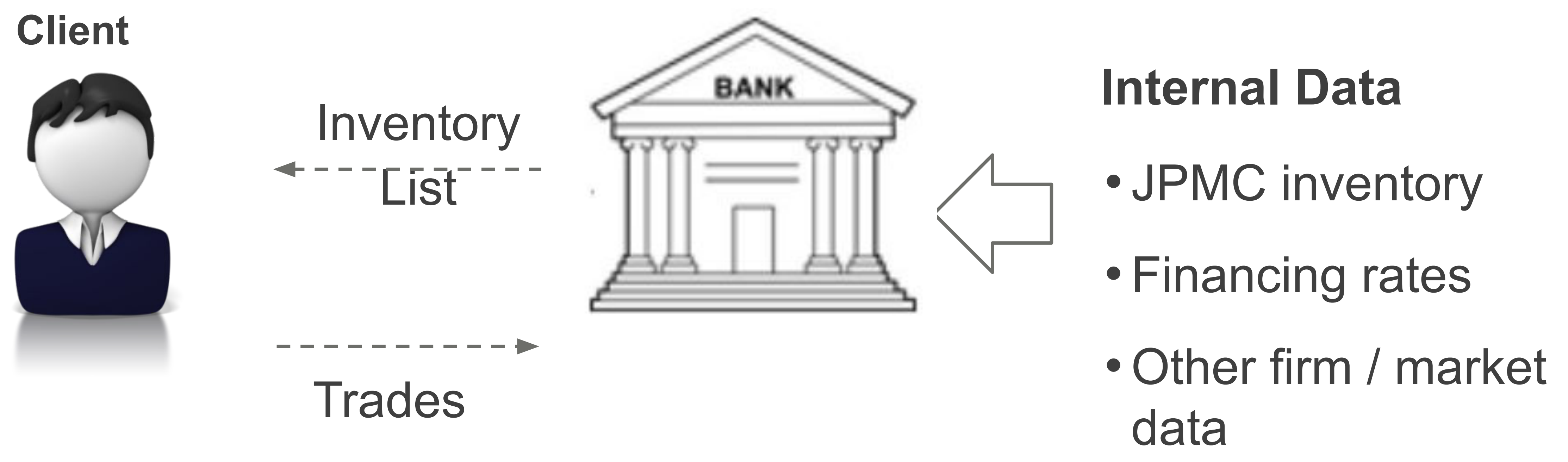
Privacy-preserving inventory matching at J.P. Morgan

[Polychroniadou, Asharov, Diamond, et al.]

JP Morgan publishes a daily list of inventory at a discount to clients

- Based on aggregated information on previous transactions from clients

This inventory list makes it possible to offer good rates, but can leak some information about trading strategy

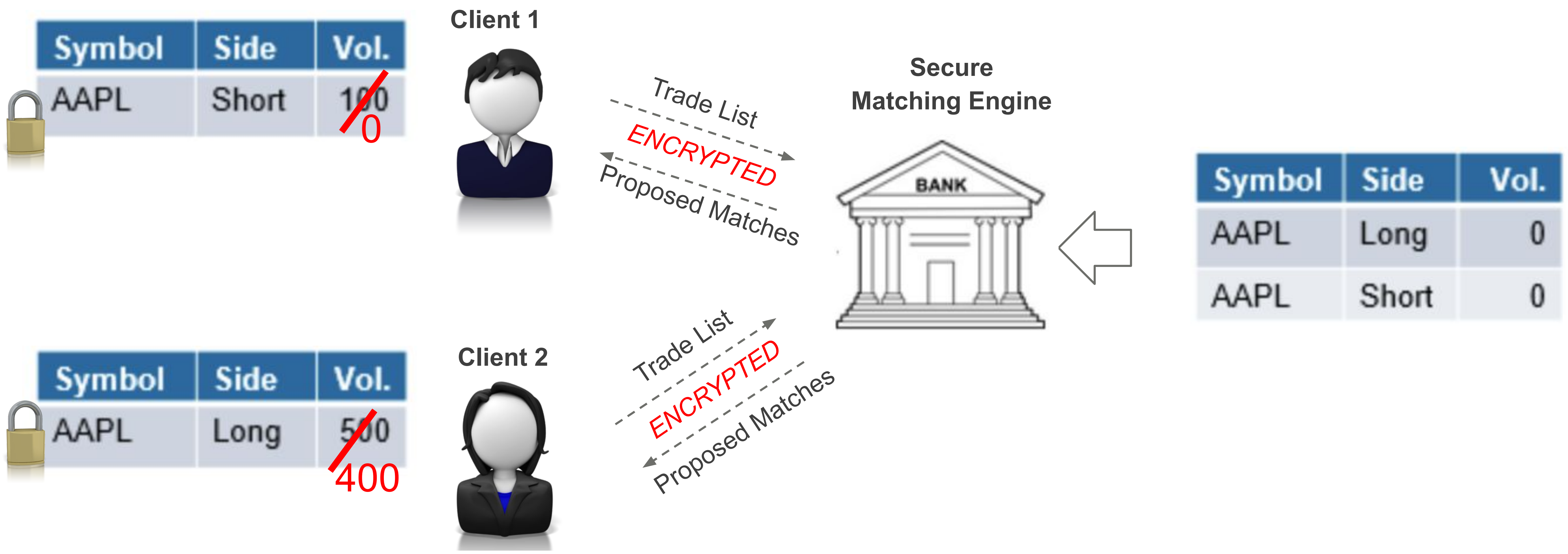


Privacy-preserving inventory matching at J.P. Morgan

[Polychroniadou, Asharov, Diamond, et al.]

Approach: Inventory matching between clients in MPC

- Clients send encrypted trade list
- Server provides full inventory list
- Match trades against other clients in MPC
- As of 2023, running in production



Cryptocurrency wallets

Fireblocks, Fordefi, Safeheron, …

Cryptocurrency wallets manage secret keys for signing user transactions

Keeping secret keys in a single location creates a single point of security failure (potentially worth millions of dollars!)

Approach: split secret key across different entities and use MPC to sign a transaction

- Secret key is never materialized in one location
- Often combined with secure hardware to offer an additional layer of protection

Genomic analysis

[Jagadeesh, Wu, Birgmeier, Boneh, Bejerano]

For medical research: Want to compare patient's genome with as many other genomes as possible

For privacy: Hide genome, as it reveals sensitive information

Approach: use MPC to find and reveal causative genetic variants while protecting remaining variants

- Helped diagnose real patients and discover previously unknown gene-disease associations
- Participants learned nothing about each other except shared disease-causing gene

Outline

1. Garbled circuits
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References

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